THE PRINCIPLE OF EXPLOSION: ARISTOTLE VERSUS THE CURRENT SYNTACTIC THEORIES

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Abstract. The principle of explosion is a problem for the syntactic theories trying to explain and describe human reasoning. In fact, most of the formal cognitive theories tend to reject it. However, that rejection is not often based on a theoretical development of the theories, but on inductions from experimental data. In this paper, I expose Woods and Irvine's arguments in order to show that Aristotelian logic does not have this problem, that its theoretical framework does not enable to accept the principle of explosion, and that this logic hence has, at least in a sense, certain advantages compared to the current reasoning syntactic theories.

Keywords: Aristotelian theory, logic, principle of explosion, reasoning, syntax.

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Introduction

The principle of explosion is usually expressed in Latin with sentences such as *ex contradictione quodlibet sequitur* or *ex falso quodlibet sequitur*, and provides that, if a contradiction is found in a logical argument, any conclusion can be drawn from that contradiction. Many logical systems accept or are based on that principle, including Gentzen's (1935) natural deduction calculus, and, of course, what is called today 'standard logic.' The problem is that the principle does not seem to be used by the human mind in a natural way. Indeed, it is not common that naïve individuals, i.e., individuals without logical knowledge background, base their

daily arguments on the principle of explosion, or that they argue in discussions or debates that, given that a contradiction has been identified, it is possible to derive any conclusion. On the contrary, the usual behavior is to derive nothing form a contradiction.

Thus, it can be said that the principle of explosion is a challenge for the systems trying to describe human reasoning, especially if such systems claim that the human mind works applying formal rules more or less akin to those of standard calculi. As indicated below, none of the current cognitive theories appear to accept the principle. However, as also shown below, some of them, although they explicitly reject it, assume at the same time a logical framework that allows drawing any sentence from a contradiction.

This is not the case of Aristotelian logic. This logic does not need to explicitly reject the principle of explosion because its theoretical framework makes it impossible. Therefore, it can be said that, while the current formal theories addressing human cognition are not systems in which the principle is actually forbidden (in general, as mentioned, such theories only claim that the principle cannot be accepted because it is obvious that people do not use it), the first logic in history does have the machinery for blocking or preventing its application. This latter idea (that the principle is not possible in Aristotelian logic) has been argued by Woods and Irvine (2004, 64-67), and, in this paper, I will expose, review, and analyze in details their theses. My main aim by doing that is to explain the relevance that such theses can have for the contemporary cognitive science and indicate what Aristotelian logic can give to the modern reasoning theories.

Thus, with these goals in mind, I will begin by explaining Woods and Irvine's (2004) theses. However, given that such theses are based in turn on other two conditions proposed by Aristotle, it seems to be opportune to analyze each of those conditions separately before exposing the theses on the principle of explosion. The next section deals with the first of such conditions.

The conclusions cannot repeat premises

Woods and Irvine (2004, 54) call the first condition '*Non-Circ*.' It is a condition that every *συλλογισμός* (syllogism) needs to fulfill and that, in short, what it establishes is that a correct *συλλογισμός* should not have one of its premises as its conclusion.

According to Woods and Irvine (2004), *Non-Circ* is a very important condition in Aristotelian logic and it is to be found in different passages of Aristotelian

texts. One of them is, for example, that of the *᾽Αναλυτικῶν Προτέρων* (*Analytica Priora*) A 1, 24^{b}, 19-20:

> "συλλογισμός δέ ἐστι λόγος εν ὧι τεθέντων τινῶν ἕτερόν τι τῶν κειμένων ἐζ ἀνάγκης συμβαίνει τῶι ταῦτα εἶναι. A *συλλογισμός* is an argument in which, if something has been said, something different from what has been said is necessarily drawn from what that (what has been said) is".

The key word in this passage is clearly *ἕτερόν*, which I have translated as 'something different from' and Woods and Irvine (2004, 51) as 'something other than'. This latter translation is, as indicated by Woods and Irvine, taken from Barnes (1984), but what is important is that both translations show clearly that, following Aristotle, a *συλλογισμός* cannot have a conclusion matching one of its premises, because what is derived from them is something undoubtedly different.

Of course, there are more examples of passages in which Aristotle defines what a *συλλογισμός* is, but I think that this one is illustrative enough and justifies, as Woods and Irvine do, to attribute *Non-Circ* to Aristotle. Thus, it can be stated that this condition means that, given this argument:

A, B / ergo Γ

It is a *συλλογισμός* only if Γ is not A and Γ is not B.

Nevertheless, to prove that the principle of explosion is not possible in Aristotelian logic, it is also necessary to take another condition into account. As shown in the next section, that condition is not really a condition, but another principle.

The principle of conversion

Indeed, the second condition actually refers to the fact that it should be possible to apply a well-known Aristotelian principle or rule to every correct *συλλογισμός*. That principle is the principle of conversion, which is very used by Aristotle in his texts.

It provides that, if a conclusion follows from two premises, then the opposite of one of the premises follows from the opposite of the conclusion and the other premise. The principle of conversion is really important in Aristotelian logic and it can be said that it is behind the demonstrations *per impossibile* (or *reduction ad absurdum* demonstrations), which, as indicated by Boger (2004, 228), are explicitly used by Aristotle in *᾽Αναλυτικῶν Προτέρων* B 11-13. So, it is obvious that the rule of conversion is an essential part of Aristotle's logic.

However, it is true that there are some discussions in this regard. As it is well known, Stoic logic includes a version of this rule, which is considered to be the first of the *θέματα* (the reduction rules) in this latter logic. The similarity between the Aristotelian and the Stoic rule is strong, and it can be checked if we take into account the following passage authored by Pseudo-Apuleius and that is to be found in *De Interpretatione* 209, 12-14:

> "Si ex duobus tertium quid colligitur, alterum eorum cum contrario illationis colligit contrarium reliquo. If a third is deduced from two (sentences), one of the two and the opposite of the third lead to the opposite of the other of the two".

Pseudo-Apuleius is speaking about Stoic logic and, as said, the similarity is clear. Nevertheless, several authors have proposed distinctions between the two rules. For example, Bobzien (1996, 144, footnote 20) claims that the Aristotelian principle referred to both contradictory and contrary elements. Nonetheless, as Bobzien also mentions (in the same footnote), Mignucci's (1993, 227-229) view is not this one. According to Mignucci, the Aristotelian rule only can be related to contradictory elements.

In any case, these details may not be very relevant for the aims of this paper, since I am more interested in the potentialities of the general Aristotelian framework and what can be derived from it than in just what Aristotle claimed and argued. Thus, what is truly worth highlighting here is that it is evident that the rule of conversion existed in Aristotelian logic, that it could be applied to any *συλλογισμός*, and that its structure was akin to the following:

If [A, B /ergo Γ] is a *συλλογισμός*, then it can be drawn [A, ¬Γ / ergo ¬B].

Where '¬X' represents the opposite of 'X'. Of course, by paying attention to the discussion between Bobzien and Mignucci, different interpretations on what 'the opposite of X' means can be raised. It can be said, for example, that it means 'the contrary of X', 'the contradictory of X', or both of them. However, if the goals of this paper are taken into account, the only point that should be considered is that X and ¬X are incompatible sentences that cannot be accepted at the same time.

The principle of explosion is not possible in Aristotelian logic

In this way, based on *Non-Circ* and the rule of conversion, Woods and Irvine (2004, 64-67) demonstrate that, in Aristotelian system, it cannot be stated that '*ex falso quodlibet sequitur*', i.e., that any conclusion follows from the false. Their proof is more or less this one:

According to *Non-Circ*, it cannot be accepted that this argument is a *συλλογισμός*:

A, B /ergo B.

So, if we apply the rule of conversion to the latter argument, we will get a new argument that cannot be considered to be a *συλλογισμός* either. And, obviously, a possible use of the rule can be to transform the previous argument into the following:

B, ¬B / ergo ¬A

Thus, given that this is not a *συλλογισμός*, it is absolutely clear that nothing can be deduced from two incompatible premises, and that the principle of explosion hence is not possible in Aristotelian logic.

In connection with this, Woods and Irvine (2004, 64) see clear relations between Aristotelian logic and Bolzano's (1837) logical system, since the principle of explosion is not admitted in this latter system either. And this in turn leads them to propose that Aristotelian logic can be thought to be the very "first paraconsistent logic" (Woods & Irvine 2004, 65). In fact, they think that Aristotle considered his principles and restrictions to be absolutely necessary because he was trying to describe how individuals actually reason in their everyday life (Woods & Irvine 2004, 66).

Despite this, it does not seem to be appropriate to say that Aristotle's logic is the logic that better describe how human reasoning works, because there are many aspects involved in reasoning that it does not appear to have taken into account systematically (e.g., probabilities or temporal relations). On the other hand, as indicated below, there are also many theories with strong empirical support that can fairly accurately explain, and even predict, the human inferential activity. Nevertheless, Aristotelian logic has a characteristic in this way that deserves to be highlighted and acknowledged. If Woods and Irvine (2004) are right, it can be considered to be one of the few proposals trying to show the real way in which the human mind works that is syntactic or formal and, at the same time, has the necessary machinery to block or forbid theoretically the use of the principle of explosion. I explain this idea in more details in the next section.

The contemporary reasoning theories and the principle of explosion

As said, there are several theories addressing human reasoning today. Such theories are based on very different assumptions and suppositions, and their approaches hence are not, in many cases, very akin. Two important examples can be the probability logic theory (e.g., Adams 1998; Adams & Levine 1975; Oaksford & Chater 2009; Pfeifer 2012, 2015), which, in general, claims that human reasoning is not linked to standard logic, but based on the analysis of the probabilities of the events involved in the inferences, and the mental models theory (e.g., Johnson-Laird 2004, 2006, 2012, 2015; Oakhill & Garnham 1996), which shares with the previous one that the human mind does not follow the formal rules of classical logic to make inferences, but it proposes another alternative: individuals come to conclusions by considering the semantic possibilities (mental models) that correspond to the sentences included in arguments and describe the different scenarios consistent with such sentences. Theories such as these ones do not often have problems with the principle of explosion. Given that their approaches come from frameworks other than standard logic, difficulties such as those that the principle raises make no sense in them and are extremely unlike.

But the case of the more or less syntactic or formal theories based on calculi such as, e.g., Gentzen's (1935) natural deduction calculus is different. As it is well known, Gentzen's calculus allows using the principle of explosion and, of course, if we wish to argue that that calculus describes the human inferential activity, this is a problem that needs to be solved. However, the truth is that, at the present time, it is very hard to find syntactic theories holding that the behavior of the human mind can be explained by just Gentzen's (1935) system. The current theories, although they do not ignore all of the formal rules proposed by Gentzen, are usually based on empirical experimentation and, for this reason, tend to reject the rules of Gentzen's calculus that, according to experimental evidence, people appear not to apply. Thus, it can be said that it is very difficult to find today a theory claiming that individuals can use the principle of explosion in their reasoning processes. And this is so because, as indicated, the empirical results show that naïve people (that is, people without background on logic) do not generally consider the principle.

But this does not mean that, in addition to the empirical rejection, all of the formal theories have the theoretical tools to explain why individuals do not resort to the principle of explosion. Although there are several contemporary syntactic theories, I will only focus on one of them here, the mental logic theory (e.g., Braine & O'Brien 1998a; O'Brien 2009, 2014; O'Brien & Li 2013; O'Brien & Manfrinati 2010). The reason is that this theory and its problems with the principle of

explosion can be illustrative enough and, as far as I know, what I will expose below on the mental logic theory can be easily said in the same way about other formal theories.

The case is that, the mental logic theory, as indicated on the syntactic theories in general above, does not accept all of the formal rules of standard logic or Gentzen's (1935) formal calculus. However, to indicate which of those rules the theory admits and which of them it rejects does beyond the purposes of this paper. What really interests to us here is how the mental logic theory deals the principle of explosion. Obviously, because, as also said on the current formal theories, this theory considers empirical evidence, it cannot accept that '*ex falso quodlibet sequitur*.' Contradictions or, better yet, incompatibilities play a role in its framework. Nevertheless, that role refers to the *reduction ad absurdum* strategy, not to the principle of explosion. In this way, the appearance of an incompatibility in an inferential process only leads individuals to think that at least one of their assumptions is not true, and not to derive any conclusion (Braine & O'Brien 1998b, 206).

Nonetheless, in my view, the problem is not solved with that. As in the case of other syntactic theories, this rejection of the principle of explosion comes only from the empirical data, which inform that people do not actually use it. In this regard, it can be said that the argument is only inductive, and that it is only a generalization of experimental results. Given that it is observed that individuals do not tend to use the principle of explosion, it is said that that principle is not a part of the human mental logic. Therefore, the problem is the one indicated in general above: the theoretical framework of the theory does not prevent its use. Unlike Aristotelian logic, the mental logic theory does not have resources such as *Non-Circ* or the rule of conversion that forbid or block its application. Therefore, the reason of the rejection is not truly demonstrated by this latter theory. It is only an assumption of it, and not a consequence of its theses.

Furthermore, if we consider just the general theses of the mental logic theory, we can realize that they really allow the use of the principle. Let us suppose that A and B are assumptions in an inference, and that, after a number n of steps applying formal rules admitted by the mental logic theory, we come to a scenario such as this one:

[1] A	(assumption)
[2] B	(assumption)
...	
[n-1] Γ	(...)
[n] ¬Γ	(...)

Steps [n-1] and [n] inform that an incompatibility exits and that at least one of the assumptions, in this case [1] and [2], or both of them, is not correct. But the problem here is that we do not know which the wrong assumption(s) is(are). Is it necessary to remove A? B? Both of them? As far as I know, the theory cannot respond to these questions, since it does not include a procedure or program to detect or identify the assumptions that should be eliminated when an incompatibility is found.

And this unsolved problem is what enables to use the principle of explosion. Indeed, there is nothing that prevents that we add one more assumption, with the content that we wish, to the previous deduction. Thus, we could add a step [o] with, for example, the assumption Δ, and, given that steps [n-1] and [n] reveal that there is an incompatibility, and that at least one of the our assumptions is wrong, we could undoubtedly conclude ¬Δ (as indicated, the theory does not give rules or procedures for making a decision about which the assumption to be removed is).

This is clearly the use of the principle of explosion. It can be applied in the mental logic theory, and the reason is, as said, that its rejection of the principle is only empirical and inductive. So, Aristotelian logic has something that the mental logic theory does not: a theoretical framework within which it can be demonstrated that the principle of explosion cannot be used.

Conclusions

The previous pages show that some of the current theories on cognition have certain problems that Aristotelian logic does not. So, it can be thought that, if ancient logics could solve such problems, the current theories must do that as well. An interesting consequence of all of this is that it makes clear that ancient logics should not be ignored or forgotten. It is evident that the contemporary theories better explain mental processes, but it is also so that the ancient theories can be very useful today too, since they can provide ideas and clues to face some difficulties.

Thus, the fact that it was proposed many centuries ago and that it is not an empirical theory can lead one to think that Aristotelian logic is obsolete and outdated. However, the precedent arguments indicate that it is obvious that Aristotle's theory of the *συλλογισμός* also have something to offer. And this is so for several reasons, but the one that has been analyzed in this paper is that Aristotelian logic eliminates a very important difficulty that some syntactic or formal theories appear to continue to have.

As explained, the mental logic theory rejects the principle of explosion, and it is absolutely necessary to value its arguments on the role that incompatibilities play in human reasoning. Nevertheless, what one would expect from it is that its rejection were not only empirical, but supported on theoretical bases as well, as, for example, following Woods and Irvine (2004), it is the case in Aristotelian logic.

By this I do not mean (and maybe it is important to insist in this idea) that Aristotelian logic is a clear alternative to the mental logic theory for describing or predicting human reasoning. In fact, it seems that, at present, neither Aristotelian logic nor the mental logic theory are the theories with more empirical support. The experimental results that are to be found in the literature on cognitive science appear to give a relevant advantage to other theory cited above, the mental models theory. Thus, if the mental logic theory wishes to become a real option deserving to be considered and different from the mental models theory, it needs to improve certain aspects. On of them is that studied in this paper, and, as far as this issue is concerned, my only claim is that the resources of Aristotelian logic can be very useful for that work.

Furthermore, it can be said that this particular case reviewed here makes explicit the sense and the validity that the theories presented in the past may have today. And this applies not only to Greek logic, but also to ancient philosophy and science in general, including, of course, those of all of the traditions and cultures.

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